Standard Errors: Statistical Consequences of Health Care Provider Insurance Risk Assumption

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Why does insurance work?

The Central Limit Theorem - Just Like Sampling Theory
- Insurers randomly select policyholders within rating categories
- Portfolio loss ratios estimate population loss ratios
- Actuarial task:
  - Predict future population loss ratio
  - Determine expense, risk, and profit loaded premium
  - Establish and test reserve and solvency adequacy

Neglected issue - How risk disaggregation/small portfolio size affects:
- Profits
- Losses
- Surplus requirements
- Insolvency risk
- Policyholder benefit levels
Health Care Provider (HCP) Insurance Risk Assumption

HCPs become insurers through:

- Global and partial capitation
- 3rd party payer/Provider profit/risk sharing agreements
- Bundled/Episode based payments
- Diagnosis Related Groups (DRGs)
- Medicare/Medicaid Prospective Payment Systems (PPS)

Flaws in provider risk assumption

- HCPs become small insurers
- Small insurers less efficient risk managers than large insurers
- HCPs must prevent things that may never happen
- HCPs responsible for correlated & un-correlated effects of Dx & Tx
- Decreased not increased HCP efficiency
- Decreased health care service quality and quantity
Paradigm Insurer issues 1,000,000 policies
Premium = $4,000/year/policyholder
Expected loss ratio = 0.7500
Portfolio loss ratio estimate (PLRE) standard error ($\sigma_{e_{PL}}$) = 0.0500
Profit margin = 0.0500
Expense ratio = 0.1500

Portfolio loss ratio estimate variation ($\sigma_{e_{I}}$) determines:
- Profits
- Losses
- Insolvency
- Solvency standards
- Surplus requirements
- Policyholder benefit levels
Modeling Insurer Loss Ratios

**Characteristics of Individual Policyholder Loss Ratios**

- $\mu = 0.7500$ (Average loss ratio = $\frac{\$3,000 \text{ losses}}{\$4,000 \text{ premium}}$)
- Variance = $\sigma = 50.0000$
- Many $0$ claims
- Policyholders randomly selected

**Characteristics of Insurer Loss Ratios**

- $\mu = 0.7500$ (Average loss ratio = $\frac{\$3,000 \text{ losses}}{\$4,000 \text{ premium}}$)
- $\sigma_{eN} = \frac{50.0000}{\sqrt{N}}$ ($\sigma_{ePI} = \frac{50.0000}{\sqrt{1,000,000}} = 0.0500$)
- Many policyholders - smallest insurer 10,000
- Insurer loss ratios assumed to be $N(\mu, \sigma_{eN})$
- Insurers *must* hold surplus up to $\mu + 3 \times \sigma_{eN}$ ($P[\text{Insolvency}] = 0.0013$)
Paradigm Insurer Operating Characteristics

\[
\begin{align*}
\text{P}[\text{Profit} \geq 10\%] &= 0.5000 \\
\text{P}[\text{Profit} \geq 5\%] &= 0.8413 \\
\text{P}[\text{Profit} \geq 0\%] &= 0.9772 \\
\text{P}[\text{Losses} \geq 0\%] &= 0.0228 \\
\text{P}[\text{Losses} \geq 5\%] &= 0.0013 \\
\text{P}[\text{Losses} \geq 10\%] &= 0.0000
\end{align*}
\]

Solvency Preserving Loss Ratio = 0.9000
Surplus required = $200,000,000

\[
\begin{align*}
\text{MSB}_{\text{Profits} \geq 5\%} &= $0.7500 \\
\text{MSB}_{\text{Profits} \geq 5\%} &= $3,000
\end{align*}
\]
## Insurer Operating Results By Portfolio Size

<table>
<thead>
<tr>
<th>Size (1,000s)</th>
<th>NHI</th>
<th>B</th>
<th>PI</th>
<th>D</th>
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<tbody>
<tr>
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<td>1,000</td>
<td>100</td>
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<td></td>
</tr>
<tr>
<td>$\sigma_{e_{size}}$</td>
<td>0.00285</td>
<td>0.01581</td>
<td>0.0500</td>
<td>0.1581</td>
<td>0.5000</td>
</tr>
<tr>
<td>P[Profit $\geq$ 10%]</td>
<td>0.5000</td>
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<td>P[No loss]</td>
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<td>0.7365</td>
<td>0.5793</td>
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<td>SPLR</td>
<td>0.7586</td>
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<td>0.9000</td>
<td>1.2243</td>
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<tr>
<td>Surplus Needs $1M</td>
<td>$0</td>
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<td>$200</td>
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<td>Tot Surplus $1M</td>
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### MSB_{Profits $\geq$ 5%}
- NHI: $0.7972$
- B: $0.7842$
- PI: $0.7500$
- D: $0.6419$
- E: $0.3000$

### MSB_{Profits $\geq$ 5%} $\quad$ $\quad$ $\quad$ $\quad$ $\quad$ $\quad$
- NHI: $3,189$
- B: $3,137$
- PI: $3,000$
- D: $2,568$
- E: $1,200$

### MSB_{No loss}
- NHI: $0.8443$
- B: $0.8184$
- PI: $0.7500$
- D: $0.5338$
- E: $0.0000$

### MSB_{No loss} $\quad$ $\quad$ $\quad$ $\quad$ $\quad$
- NHI: $3,377$
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- PI: $3,000$
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<th>Operating Profit</th>
<th>Operating Profit = Premiums − Expenses − Losses (1)</th>
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<td>Premiums exceed expenses and losses</td>
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<tr>
<th>Operating Losses</th>
<th>Operating Losses = Premiums − Expenses − Losses (2)</th>
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<td>Expenses and losses exceed premiums</td>
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<th>Insolvency</th>
<th>Insolvency = (Premiums + Surplus) − (Expenses + Losses) (3)</th>
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Population Characteristics

Individual policyholder

- Premium = $4,000 per year
- Expected Loss Ratio = 0.7500
- Standard Deviation $\sigma = 50.0000$
Paradigm Insurer Characteristics

Operating Profit

- Premium = $4,000 per person, per year
- E[PLRE] = 0.7500 (75% of premiums goes to losses)
- Y-Y variation in PLRE (Standard Error $\sigma_{e_{1,000,000}} = 0.0500$)
- Profit margin in premium = Risk Premium = $\sigma_{e_{1,000,000}}$

When premiums exceed expenses and losses

Operating Losses

Operating Losses = Premiums − Expenses − Claims (4)

When expenses and losses exceed premiums
Why Standard Errors?

Insurer population loss ratio (PLR) variation is a function of:
- Individual variation
- Insurer portfolio size

Risk disaggregation (Insurer sub-portfolio transfers to health care providers) yields less accurate estimates of population loss ratios
- Decreased accuracy of estimators/estimates
- Decreased efficiency of estimators/estimates
Risk disaggregation yields less accurate estimates of population loss ratios and this means:

- Lower probabilities of modest profits
- Higher probabilities of modest losses
- Much higher probabilities of severe losses
- Lower Maximum Sustainable Policyholder Benefits
- Higher surplus requirements
Insurer Operating Results

Insurers operating results are primarily influenced by variations in loss ratios

\[
\text{Profit (Loss)} = \text{Premiums} - \text{Expenses} - \text{Claims}
\]  

(6)

\[
\text{Insolvency} = \text{Premiums} - \text{Expenses} - \text{Claims} - \text{Surplus}
\]  

(7)
The flaw in capitation: Assuming the risks for large and small insurers are identical, when risks vary with portfolio size. Large and small insurers and risk assuming health care providers would have identical risks if standard deviations measured risk. But standard errors are identical independent of portfolio size.

*Standard errors*, not standard deviations, must be used to analyze the impact of capitation like mechanisms. I will analyze insurer performance risks, probabilities of profits, losses, insolvency, and benefit levels as they vary by portfolio size. I will also show that small insurers require far higher surplus and that this mitigates against provider insurance risk assumption.
If $\sigma^2$ is the variance for the loss ratio of an individual policyholder, the standard error for a portfolio of $n$ randomly selected policyholders, $\sigma_{en}$, is:

$$\sigma_{en} = \sqrt{\frac{\sigma^2}{n}}$$  \hspace{1cm} (8) \\

The standard errors, loss ratio probability distributions, and probabilities of specific operating results for any two insurer’s, M and N, writing portfolios of size $m$ and $n$ are not equal unless $m = n$. 

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We will look at the following operating outcomes
Profitability 10%, 5%, 0% of premium revenues
Losses 0%, 5%, 10% of premium revenues
Solvency Preserving Loss Ratio The flaw in capitation: Assuming the risks for large and small insurers are identical, when risks vary with portfolio size. Large and small insurers and risk assuming health care providers would have identical risks if standard deviations measured risk. But standard errors are identical independent of portfolio size.

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I will analyze insurer performance risks, probabilities of profits, losses, insolvency, and benefit levels as they vary by portfolio size.

I will also show that small insurers require far higher surplus and that this mitigates against provider insurance risk assumption.
General form for the $F$ distribution with $\nu_1$ and $\nu_2$ degrees of freedom:

$$ f(F; \nu_1, \nu_2) = \frac{\Gamma \left( \frac{\nu_1+\nu_2}{2} \right)}{\Gamma \left( \frac{\nu_1}{2} \right) \Gamma \left( \frac{\nu_2}{2} \right)} \left( \frac{\nu_1}{\nu_2} \right)^{\frac{\nu_1}{2}} \frac{F^{\frac{\nu_1-2}{2}}}{\left[ 1 + \left( \frac{\nu_1}{\nu_2} \right) F \right]^{\frac{\nu_1+\nu_2}{2}}} $$
Solvency Preserving Loss Ratio

What is the Solvency Preserving Loss Ratio

- All insurers should protect against losses greater than expected
- Exposure
- Differences in perceived and actual risks
- Consequences of differences in exposure to risk
- Premium and surplus adequacy varies by size
- Cost of money varies by size
- Stockholder/Policyholder expectations

Maximum Sustainable Benefits

- Keyed to other performance characteristics:
  - Profit goals
  - Loss aversion
  - Insolvency aversion
  - Return on investment
Calculating Prospective Policyholder Benefits

- Claim settlement practices vary by insurer
  - Portfolio size differences
  - Differences in perceived and actual risks
  - Consequences of differences in exposure to risk
  - Premium and surplus adequacy varies by size
  - Cost of money varies by size
  - Stockholder/Policyholder expectations

Maximum Sustainable Benefits

- Keyed to other performance characteristics:
  - Profit goals
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  - Return on investment
Policyholder Benefits Vary By Portfolio Size

**Maximum Sustainable Benefits for Profits ≥ 5%**

- Insurer probabilities of profits ≥ 5% vary by portfolio size
- Insurers seek PI’s profit profitability: \( \text{P}[\text{Profits}_{PI} \geq 5\%] = 0.8413 \)
  - Have different standard errors and PLRE variation
  - Must target benefits to different levels or risk missed profit goals, losses, and insolvency
  - Adjust claims settlement practices to meet profit goal
  - Insurers should target benefits one standard error below the maximum loss ratio (0.8000) consistent with profits ≥ 5%

**Quick and Dirty Maximum Sustainable Benefits**

\[
MSB_I = \text{Maximum Loss Ratio for Profits } \geq 5\% - 1 \times \sigma_{e_I} \quad (10)
\]

\[
MSB_I = 0.8000 - 1 \times \sigma_{e_I} \quad (11)
\]
Policyholder Benefits Vary By Portfolio Size

**Maximum Sustainable Benefits To Avoid Losses**

- Insurer probabilities of losses vary by portfolio size
- Insurers seek PI’s loss avoidance probability: \( P[Losses_{PI} \geq 0\% = 0.0228] \)
  - Have different standard errors and PLRE variation
  - Must target benefits to different levels or risk missed profit goals, losses, and insolvency
  - Adjust claims settlement practices to avoid losses
  - Insurers should target benefits two standard errors below the maximum loss ratio (0.8500) consistent with avoiding losses \( \geq 0\% \)

**Quick and Dirty Maximum Sustainable Benefits**

\[
MSB_I = \text{Maximum Loss Ratio To Avoid Losses} \geq 0\% - 2 \cdot \sigma_{e_I} \quad (12)
\]

\[
MSB_I = 0.8500 - 2 \cdot \sigma_{e_I} \quad (13)
\]